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Stable Plithogenic Cubic Sets

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Abstract

In this article, the ideas of a stable plithogenic cubic set, stable plithogenic element, evaluation Plithogenic Set (PS), and stable plithogenic degree for fuzzy, intuitionistic and Neutrosophic Sets (NSs) are introduced, and associated properties are examined. Concerning internal (external) Plithogenic Neutrosophic Cubic Sets (PNCSs) and the complement of PNCSs, their stableness and unstableness are discussed. Regarding the P-union, R-union, P-intersection and R-intersection of PNCSs, their stableness and unstableness are studied with numerical examples.

Keywords: Plithogenic evaluative set, Plithogenic evaluative point, Stable cut, Stable element, Unstable cut.

1 | Introduction

In 1965, Zadeh [1] developed the concept of Fuzzy Sets (FSs), breaking with the idea of traditional set theory, believing that objects have a certain degree of ambiguity and that exploiting ambiguities might more properly convey many difficulties to describe problems in real life. The membership function serves as the foundation of FS theory, which is a generalization of classical set theory. It represents the degree to which an element belongs to a set. The membership function has a value in the range [0,1]. The closer the value is to one, the greater the degree of membership. In general, FSs are more abstract, but their membership is a definite real number; thus, FSs have considerable advantages in real life and are commonly employed. Since then, numerous studies have expanded the research on FSs based on their advantages. Because of the problem's complexity, the FS theory has some flaws in the problem-solving process. As a result, some researchers proposed many extended versions of FSs, such as Intuitionistic FSs (IFSs), Interval-Valued Intuitionistic FSs (IVIFSs), and Hesitant FSs (HFSs), which have been utilized to handle numerous uncertain situations.

Atanassov [2] introduced IFSs, which consider not only the membership degree of an element of the set but also the non-membership degree of an element and are commonly utilized. The membership and non-

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membership degrees of elements in IVIFSs were increased from a numerical value to a subset of the interval [0,1]. These expanded FSs solved several practical difficulties and drew an increasing number of scientists to study them.

Smarandache [3] presented Neutrosophic Sets (NSs) as a valuable tool for solving uncertain and inconsistent situations. In solving practical problems, NSs take into account not only the membership and non-membership degrees of elements but also their ambiguity and inconsistency. In contrast to FSs and IFSs, NSs express the incompleteness and inconsistency of an element to set, making them better suited to solving actual problems. As a result, numerous expanded versions of NSs, such as Single-Valued Neutrosophic Sets (SVNSs) and Interval-valued Neutrosophic Sets (INSs), have been proposed and used to solve many practical situations.

Jun et al. [4] proposed the notion of Neutrosophic Cubic Sets (NCSs), which defined internal and exterior NCSs and discussed their related aspects by extending cubic sets to NCSs. Also defined P, R-union and P, R-intersection of neutrosophic cubic set.

The genesis, origination, creation, formation, development, and evolution of new entities from dynamics and organic fusions of contradictory neutral and non-contradictory multiple old entities is the theory of plithogeny developed by Smarandache [5–7] in 2017. At the same time, plithogenic refers to everything related to plithogeny.

The study by Saraswathi et. al [8] focuses on developing a decision-making approach for analyzing Fuzzy Relational Maps (FRMs) under conditions of uncertainty. It emphasizes the utility of FRMs in modeling complex systems with ambiguous and interrelated factors. Using a case study approach, the authors highlight how their method identifies key causal relationships and evaluates their effects. This provides valuable insights into optimizing decision-making processes, particularly in contexts characterized by vagueness and uncertainty. The research contributes to advancing systemic analysis methodologies and offers practical implications for addressing real-world problems in various domains using fuzzy logic techniques.

Montazeri et al. [9] introduce a robust-fuzzy-probabilistic optimization model for resilient, sustainable supply chains, focusing on seller's inventory management. Their approach integrates robust fuzzy programming to handle uncertainties, enhancing supply chain resilience and sustainability. Sallam et. al [10] utilize single valued neutrosophic sets to assess supplier quality in uncertain environments, enhancing decision-making in supply chain management.

Fujita et. al [11] introduced local-neutrosophic logic and local-NSs, extending traditional neutrosophic logic by incorporating locality. This framework enhances the modeling of spatially dependent uncertainty, offering improved applications in fields such as image processing and pattern recognition. Their approach addresses limitations in existing neutrosophic systems by accounting for the local context.

A Plithogenic Set (PS) P is a set whose elements are distinguished by one or more attributes, each of which may have several values. Concerning some stated criterion, each attribute's value v has a corresponding degree of appurtenance d(x,v) of the element x to the set P. A dissimilarity measure is defined between each attribute value and the dominant (most important) attribute value to improve the accuracy of the plithogenic aggregation operators. However, there are times when such a Dominant Attribute Value (DAV) is not taken into account or does not exist; therefore, it is assumed to be zero by default, or there are many DAVs. In such circumstances, either suppress the dissimilarity measure function or build another relationship function between attribute values.

The plithogenic aggregation operators (intersection, union, complement, inclusion, and equality) are based on the degree of contradiction between the values of the attributes. The first two are linear combinations of the t-norm and t-conorm of the fuzzy operators. The PS is a generalization of the crisp set, FS, intuitionistic FS, and NS because these four types of sets are distinguished by a single attribute value (appurtenance), which has one value (membership) for the crisp and FSs, two values (membership and non-membership) for the intuitionistic FS, and three values (membership, non-membership and indeterminacy) for the NS.

Priyadharshini et al. [12] introduced a comprehensive exploration of plithogenic and cubic sets, providing a solid theoretical foundation and introducing innovative concepts like plithogenic fuzzy, intuitionistic fuzzy, and NCSs. It extends these ideas to topological spaces, ensuring a broad application range. Their properties add depth to the study. Practical applications in multi-criteria decision-making, obesity analysis, and agriculture demonstrate the real-world relevance of these concepts. The approach of the thesis simplifies complex computations, making it accessible to researchers and practitioners dealing with uncertain and conflicting data.

Kumar et. al [13] present an analytical approach to the fractional Swift-Hohenberg equation incorporating uncertainty. They employ computational algorithms to address numerical challenges, providing insights into the equation's behavior under uncertain conditions. Their findings contribute to the understanding and application of fractional differential equations in complex systems.

Nivetha et al. [14] introduced Plithogenic Cognitive Maps (PCM), a novel decision-making tool that incorporates the concept of contradiction degrees. It extends previous models like Fuzzy Cognitive Maps (FCM) and Neutrosophic Cognitive Maps (NCM) by integrating plithogenic operators to handle indeterminacy in decision-making better. The paper offers a systematic methodology for applying PCMs and demonstrates their effectiveness through real-world applications, particularly in agriculture. The approach is highly adaptable, with potential extensions to various forms, such as interval-valued PCMs, and presents a powerful tool for analyzing complex decision problems involving uncertainty.

Alkhazaleh [15] combines soft set theory with PS theory to handle complex uncertainties and contradictions in decision-making processes. The merits of this paper include its innovative approach to managing multi-criteria decision-making environments, its application to fuzzy, intuitionistic fuzzy, and NSs and the introduction of operations such as union intersection for plithogenic soft sets. Theoretical concepts are supported with examples, providing a valuable framework.

Singh [16] formulated the concept designed to extend traditional, fuzzy, and NSs to better handle complex situations involving uncertainty, especially in cases with multi-valued attributes. This approach can be particularly useful for researchers working in data analysis and neutrosophic studies, where uncertainty and inconsistency are common challenges. By offering an initial method for establishing PSs, the author provides a foundation that other researchers can build upon for further work in managing uncertainty in various applications.

This research work aims to explore the behavior and core properties of stable plithogenic cubic sets under specific ordering rules (P and R order), which can be significant for applications where uncertainty and multi-dimensional data are involved. This can be useful in fields like decision-making, artificial intelligence and data analysis.

2 | Preliminaries

Definition 1. A PS is a generalization of a crisp set, a FS, an Intuitionistic Fuzzy Set (IFS), and a NS. At the same time, these four categories are represented by a single attribute value (appurtenance): a single value (belonging)-for a crisp set and an FS, two values (belonging, non-belonging)-for an IFS, or triple values (belonging, non-belonging and indeterminacy) for NS [7]. In general, PS is a set whose members are determined by a set of elements with four or more values of attributes.

Definition 2. Let Z be the universal set. A non-empty set $B=\{\beta_1,\beta_2,...\beta_s\},s^31$ of uni-dimensional parameters and $\beta \in B$ attributes is known as the attribute value continuum of the PS [7]. A given value whose range of all possible values (or states) is the non-empty set U is any finite discrete set $U=\{u_1,u_2,...,u_s\},1\leq s<\infty$, infinitely countable set $U=\{u_1,u_2,\dots,u_\infty\}$ or infinitely uncountable (continuum) set $U=]x,y[, x<y, \text{where}]\dots[$ is any open, quasi-open or closed interval from the set of real numbers of another general set.

Definition 3. Generally, there is a DAV within the value set R of the attribute, which is defined by the experts upon their application. Dominant value means the most significant value of the attribute in which the experts are involved [7]. There are situations where such a DAV may not be taken into consideration or does not exist, or several dominant (essential) attribute values may exist when various methods would be applied.

Definition 4. Let the cardinal $|R| \geq 1$. Let $C: R * R \rightarrow [0,1]$ be the attribute value contradiction (dissimilarity) degree function between any two attribute values r_1 and r_2 denoted by $C(r_1, r_2)$, which satisfies the following conditions [7]:

I. $C(r_1, r_2) = 0$, the contradiction (dissimilarity) degree between the same attribute values is zero;

II. $C(r_1, r_2) = C(r_2, r_1)$ commutativity.

Definition 5. Let $\Lambda = \langle A, \lambda \rangle$ be a cubic set in a non-empty set X. Then the evaluative set of $\Lambda = \langle A, \lambda \rangle$ is defined to be a structure $\Xi_\Lambda = \{(x, E_\Lambda(x)) \mid x \in x\}$, where $E_\Lambda(x) = \langle l(E_\Lambda(x)), r(E_\Lambda(x)) \rangle$ with $l(E_\Lambda(x)) = \lambda(x) - A(x)^-$ and $r(E_\Lambda(x)) = A(x)^+ - \lambda(x)$ which are called the left evaluative point and the right evaluative point respectively of $\Lambda = \langle A, \lambda \rangle$ at $x \in X$. We say that $E_\Lambda(x)$ is the evaluative point of $\Lambda = \langle A, \lambda \rangle$ at $x \in X$ [4].

Definition 6. Let $\Lambda = \langle A, \lambda \rangle$ be a cubic set in X with the evaluative set $\Xi_\Lambda = \{(x, E_\Lambda(x)) \mid x \in x\}$. An element $a \in X$ is called a stable element of $\Lambda = \langle A, \lambda \rangle$ in X if it satisfies $l(E_\Lambda(a)) = \lambda(a) - A(a)^- \geq 0$, $r(E_\Lambda(a)) = A(a)^+ - \lambda(a) \geq 0$. Otherwise, we say that it is an unstable element $\Lambda = \langle A, \lambda \rangle$ in X. The set of all stable elements of $\Lambda = \langle A, \lambda \rangle$ in X is called the stable cut of $\Lambda = \langle A, \lambda \rangle$ in X and is denoted by S_Λ. The set of all unstable elements of $\Lambda = \langle A, \lambda \rangle$ in X is called the unstable cut of $\Lambda = \langle A, \lambda \rangle$ in X and is denoted by U_Λ. We say that $\Lambda = \langle A, \lambda \rangle$ is a stable cut $S_\Lambda = X$. Otherwise, $\Lambda = \langle A, \lambda \rangle$ is called an unstable cubic set [4].

Definition 7. For a non-empty set A, the Plithogenic Fuzzy Cubic Set (PFCS) is defined as $\Gamma = \{< a, X(a), \mu(a) > \mid a \in A\}$ in which A is an interval-valued plithogenic fuzzy set and μ is a FS A [14].

Definition 8. Let Δ be a universal set and A be a non-empty set. The structure $\Psi = \{< a, Y(a), \delta(a) > \mid a \in A\}$ is said to be a Plithogenic Intuitionistic Fuzzy Cubic Set (PIFCS) in A, where $Y = \{(Y_{d_i}{}^T(a), Y_{d_i}{}^F(a))\}$ is an interval-valued plithogenic intuitionistic FS in A and δ is an intuitionistic FS in Y [14].

Definition 9. Let Δ be a universal set and A be a non-empty set. The structure $\Omega = \{< a, Z(a), \lambda(a) > \mid a \in A\}$ is said to be a Plithogenic Neutrosophic Cubic Set (PNCS) in A, where $Z = \{(Z_{d_i}{}^T(a), Z_{d_i}{}^N(a), Z_{d_i}{}^F(a))\}$ there is an interval-valued plithogenic neutrosophic set in A and $\lambda = \{(\lambda_i{}^T(a), \lambda_i{}^N(a), \lambda_i{}^F(a)\}$ a NS in A [14].

The pair $\Lambda = < Z, \lambda >$ is called a PNCS over which a mapping is given $\Lambda : Z \rightarrow NC(\Delta)$. The set of all PNCSs Δ will be denoted by $P_N{}^\Delta$.

3 | Stable Plithogenic Neutrosophic Cubic Set

Definition 10. Let $\Gamma = < X, \mu >$ be a PFCS in A, then the evaluation set of $\Gamma = < X, \mu >$ is defined to be a structure $\Xi_\Gamma = \{(a, E_\Gamma(a)) \mid a \in A\}$, where $\Xi_\Gamma(a) = \langle L(E_\Gamma(a)), R(E_\Gamma(a)) \rangle$ with $L(E_\Gamma(a)) = \mu(a) - X(a)^-$ and $R(E_\Gamma(a)) = X(a)^+ - \mu(a)$ which are called the left evaluation point and the right evaluation point of $\Gamma = < X, \mu >$ at $a \in A$ respectively. We say that $\Xi_\Gamma(a)$ is the evaluation point of $\Gamma = < X, \mu >$ at $a \in A$.

Example 1. The following is an example to compute the right and left evaluation points for PFCS with the values of attribute: Intelligence Quotient (IQ), Emotional Quotient (EQ), Social Quotient (SQ), Adversity Quotient (AQ) that represents the attribute "types of intelligence".

Table 1. Evaluation point for $\Gamma =< X, \mu >$.

Dissimilarity Measure	0	0.50	0.75	1
Value of Attributes	IQ	EQ	SQ	AQ
Appurtenance Measure $X(a)$	[0.3, 0.4]	[0.5, 0.7]	[0.4, 0.7]	[0.6, 0.8]
$\mu(a)$	0.2	0.6	0.1	0.9
Evaluation Point $\Xi_\Gamma(a)$	(-0.1, 0.2)	(0.1, 0.1)	(-0.3, 0.6)	(0.3, -0.1))

Definition 11. Let $\Gamma =< X, \mu >$be PFCSs in A with the evaluation set $\Xi_\Gamma = \{(a, E_\Gamma(a)) | a \in A\}$. An element $a \in A$ is called a stable element in A if it satisfies $L(E_\Gamma(a)) = \mu(a) - X(a)^- \geq 0$ and $R(E_\Gamma(a)) = X(a)^+ - \mu(a) \geq 0$. Otherwise, we say that a is an unstable element $\Gamma =< X, \mu >$ in A. The set of all stable elements of $\Gamma =< X, \mu >$ in A is called the stable cut of $\Gamma =< X, \mu >$ in A and is denoted by SC_Γ. The set of all unstable elements of $\Gamma =< X, \mu >$ in A is called the unstable cut of $\Gamma =< X, \mu >$ in A and is denoted by USC_Γ. We say that $\Gamma =< X, \mu >$ is a stable PFCS if $SC_\Gamma = A$. Otherwise $\Gamma =< X, \mu >$ is called an unstable PFCS.

Example 2. Consider the data collected for the attribute "types of computers" as an example of the stable cut for the PFCS $\Gamma =< X, \mu >$, which satisfies the condition $L(E_\Gamma(a)) = \mu(a) - X(a)^- \geq 0$ and $R(E_\Gamma(a)) = X(a)^+ - \mu(a) \geq 0$ for the attribute values super, mainframe, mini, and personal.

Table 2. Stable cut for $\Gamma =< X, \mu >$.

Dissimilarity Measure	0	0.50	0.75	1
Value of Attributes	Super	Mainframe	Mini	Personal
Appurtenance Measure $X(a)$	[0.1, 0.5]	[0.3, 0.8]	[0.2, 0.7]	[0.1, 0.4]
$\mu(a)$	0.2	0.5	0.2	0.3
Evaluation Point $\Xi_\Gamma(a)$	(0.1, 0.3)	(0.2, 0.3)	(0.0, 0.5)	(0.2, 0.1)

Example 3. The following is an example of the unstable cut of the PFCS $\Gamma =< X, \mu >$ as the attribute values water signs and Earth signs do not satisfy the condition for a stable cut.

Table 3. Unstable cut for $\Gamma =< X, \mu >$.

Dissimilarity Measure	0	0.50	0.75	1
Value of Attributes	Fire signs	Water signs	Air signs	Earth signs
Appurtenance Measure $X(a)$	[0.3, 0.5]	[0.6, 0.8]	[0.2, 0.7]	[0.5, 0.6]
$\mu(a)$	0.4	0.3	0.4	0.2
Evaluation Point $\Xi_\Gamma(a)$	(0.1, 0.1)	(-0.3, 0.5)	(0.2, 0.3)	(-0.2, 0.4)

Definition 12. Let $\Psi =< Y, \delta >$ be a PIFCS in A, then the evaluation set of $\Psi =< Y, \delta >$ is defined to be a structure $\Xi_\Psi = \{(a, E_\Psi(a)) | a \in A\}$. Where $\Xi_\Psi(a) = \langle L(E_\Psi(a)), R(E_\Psi(a)) \rangle$ with $L(E_\Psi(a)) = \delta(a) - Y(a)^-$ and $R(E_\Psi(a)) = Y(a)^+ - \delta(a)$ which are called the left evaluation point and the right evaluation point, respectively, of $\Psi =< Y, \delta >$. We say that $\Xi_\Psi(a)$ is the evaluation point of $\Psi =< Y, \delta >$ at $a \in A$.

Example 4. Consider an example to calculate the evaluation point for the PIFCS with the following values of attribute: mild, moderate, severe, panic, which comes under the category of different levels of anxiety.

Table 4. Evaluation point for $\Psi =< Y, \delta >$.

Dissimilarity Measure	0	0.50	0.75	1
Value of Attributes	Mild	Moderate	Severe	Panic
Appurtenance Measure $Y(a)$	([0.1, 0.3], [0.3, 0.4])	([0.5, 0.7], [0.2, 0.5])	([0.6, .0.8], [0.3,0.6])	([0.1, 0.4], [0.5,0.8])
$\delta(a)$	([0.2,0.1])	([0.6, 0.1])	([0.2, 0.7])	([0.3, 0.7])
Evaluation Point $\Xi_\Psi(a)$	([0.1, 0.1], [-0.2, 0.3])	([0.1,0.1], [-0.1, 0.4])	([-0.4,0.6], [0.4, -0.1])	([0.2, 0.1], [0.2,0.1])

Definition 13. Let $\Psi =< Y, \delta >$be a PIFCS in A with the evaluation set $\Xi_\Psi = \{(a, E_\Psi(a)) | a \in A\}$. An element $a \in A$ is called a stable element in A if it satisfies $L(E_\Psi(a)) = \delta(a) - Y(a)^- \geq 0$ and $R(E_\Psi(a)) = Y(a)^+ - \delta(a) \geq 0$. Otherwise, we say that it is an unstable element $\Psi =< Y, \delta >$ in A. The set of all stable elements of $\Psi =< Y, \delta >$ in A is called the stable cut of $\Psi =< Y, \delta >$ in A and is denoted by SC_Ψ. The set of all unstable elements of $\Psi =< Y, \delta >$ in A is called the unstable cut of $\Psi =< Y, \delta >$ and is denoted by USC_Ψ. We say that $\Psi =< Y, \delta >$ is a stable PIFCS if $SC_\Psi = A$. Otherwise, $\Psi =< Y, \delta >$ is called an unstable PIFCS.

Example 5. The types of software (application, system, driver, programming) satisfies the condition $L(E_C(a)) = \delta(a) - Y(a)^- \geq 0$ and $R(E_\Psi(a)) = Y(a)^+ - \delta(a) \geq 0$, which represents that $\Xi_\Psi = \{(a, E_\Psi(a)) | a \in A\}$ is a Stable cut.

Table 5. Stable cut for $\Psi =< Y, \delta >$.

Dissimilarity Measure Value of Attributes	0 Application	0.50 System	0.75 Driver	1 Programming
Appurtenance Measure $Y(a)$	([0.1, 0.6], [0.5, 0.8])	([0.1, 0.3], [0.5, 0.9])	([0.1, 0.7], [0.2, 0.8])	([0.3, 0.6], [0.5,0.9])
$\delta(a)$	([0.2,0.7])	([0.2, 0.7])	([0.1, 0.6])	([0.4, 0.5])
Evaluation Point $\Xi_\Psi(a)$	([0.1, 0.4], [0.2, 0.1])	([0.1, 0.1], [0.2, 0.2])	([0.0, 0.5], [0.4, 0.2])	([0.1, 0.2], [0.0,0.4])

Example 6. The following is an example of an unstable cut, as the attribute values 12% and 18 % of GST slabs do not satisfy the conditions of the stable cut.

Table 6. Unstable cut for $\Psi =< Y, \delta >$.

Dissimilarity Measure Value of Attributes	0 5%	0.50 12%	0.75 18%	1 28%
Appurtenance Measure $Y(a)$	([0.2, 0.6], [0.6, 0.9])	([0.1, 0.4], [0.3, 0.5])	([0.5, 0.7], [0.3, 0.5])	([0.3, 0.5], [0.6,0.9])
$\delta(a)$	([0.3,0.7])	([0.6, 0.7])	([0.1, 0.2])	([0.4, 0.7])
Evaluation Point $\Xi_\Psi(a)$	([0.1, 0.3], [0.1, 0.2])	([0.5, -0.2], [0.4, 0.2])	([-0.4, 0.6], [-0.1, 0.3])	([0.1, 0.1], [0.1,0.2])

Definition 14. Let $\Lambda =< Z, \lambda >$ be a PNCS in A, then the evaluation set is defined as a structure where with $L(E_\Lambda(a)) = \lambda(a) - Z(a)^-$ and $R(E_\Lambda(a)) = Z(a)^+ - \lambda(a)$ which are called the left evaluation point and the right evaluation point of $\Lambda =< Z, \lambda >$at $a \in A$, respectively. We say that $\Xi_\Lambda(a)$ is the evaluation point of $\Lambda =< Z, \lambda >$at $a \in A$.

Example 7. The following is an example to compute the right and left evaluation point of PNCS for the attribute "types of clouds" with the attribute values citro-form., cumulo-form, strato-form, nimbo-form.

Table 7. Evaluation point for $\Lambda =< Z, \lambda >$.

Dissimilarity Measure Value of Attributes	0 Cirro-form	0.50 Cumulo-form	0.75 Strato-form	1 Nimbo-form
Appurtenance Measure $Z(a)$	([0.3, 0.4], [0.5, 0.7], [0.6, 0.8])	([0.2, 0.5], [0.1, 0.3], [0.3, 0.8])	([0.2, 0.6], [0.5, 0. 8], [0.3, 0.5])	([0.2, 0.7], [0.5,0.8], [0.3, 0.5])
$\lambda(a)$	([0.1,0.6, 0.3])	([0.3, 0.2, 0.5])	([0.7,0.2,0.1])	([0.5,0.3,0.1])
Evaluation Point $\Xi_\Lambda(a)$	([0.3, 0.5], [0.1, 0.8], [0.1, 0.9])	([0.1, 0.2], [-0.1, 0.1], [0.2, 0.3])	([0.5, -0.1], [-0.3, 0.6], [-0.2, 0.4])	([0.3, 0.2], [-0.2,0.2], [-0.2, 0.4])

Definition 15. Let $\Lambda =< Z, \lambda >$be a PNCS in A with the evaluation set $\Xi_\Lambda = \{(a, E_\Lambda(a)) | a \in A\}$. An element $a \in A$ is called a stable element of $\Lambda =< Z, \lambda >$ in A if it satisfies $L(E_\Lambda(a)) = \lambda(a) - Z(a)^- \geq 0$ and $R(E_\Lambda(a)) = Z(a)^+ - \lambda(a) \geq 0$. Otherwise, we say that it is an unstable element of $\Lambda =< Z, \lambda >$in A. The set of all stable elements of $\Lambda =< Z, \lambda >$in A is called the stable cut of $\Lambda =< Z, \lambda >$in A and is denoted by SC_Λ. The set of all

unstable elements of $\Lambda =\langle Z,\lambda\rangle$ in A is called the unstable cut of $\Lambda =\langle Z,\lambda\rangle$ in A and is denoted by USC_{Λ}. We say that $\Lambda =\langle Z,\lambda\rangle$ is a stable PNCS if $\mathrm{SC}_{\Lambda} = \mathrm{A}$. Otherwise, $\Lambda =\langle Z,\lambda\rangle$ is called an unstable PNCS.

Example 8. Consider the following values of attribute representing the criteria "types of sensors": position, pressure, temperature, and force satisfy the condition for the stable cut (i.e.) $\mathrm{L}\big(\mathrm{E}_{\Lambda}(\mathrm{a})\big) = \lambda(\mathrm{a}) - \mathrm{Z}(\mathrm{a})^{-} \geq 0$ and $\mathrm{R}\big(\mathrm{E}_{\Lambda}(\mathrm{a})\big) = \mathrm{Z}(\mathrm{a})^{+} - \lambda(\mathrm{a}) \geq 0$.

Table 8. Stable cut for $\boldsymbol{\Lambda =\langle Z,\lambda\rangle}$.

Dissimilarity Measure	0	0.50	0.75	1
Value of Attributes	Position	Pressure	Temperature	Force
Appurtenance Measure $\mathbf{Z(a)}$	([0.4, 0.6], [0.2, 0.5], [0.3, 0.9])	([0.2, 0.7], [0.4, 0.6], [0.5, 0.9])	([0.1, 0.7], [0.1, 0.6], [0.8, 0.9])	([0.3, 0.7], [0.6,0.8], [0.5, 0.7])
$\boldsymbol{\lambda(a)}$	([0.5,0.3,0.4])	([0.4,0.4, 0.7])	([0.7,0.3,0.8])	([0.4,0.7,0.5])
Evaluation Point $\boldsymbol{\Xi_{\Lambda}(a)}$	([0.1, 0.1], [0.1, 0.2], [0.1, 0.5])	([0.2, 0.3], [0.0, 0.2], [0.2, 0.2])	([0.6, 0.0], [0.2, 0.3], [0.0, 0.1])	([0.1, 0.6], [0.1,0.1], [0.0, 0.2])

Example 9. The values of attributes frictional, structural, cyclical and seasonal represent the "reasons for unemployment," which is an example of an unstable cut.

Table 9. Unstable cut for $\boldsymbol{\Lambda =\langle Z,\lambda\rangle}$.

Dissimilarity Measure	0	0.50	0.75	1
Value of Attributes	Frictional	Structural	Cyclical	Seasonal
Appurtenance Measure $\mathbf{Z(a)}$	([0.5, 0.6], [0.1, 0.3], [0.4, 0.8])	([0.3, 0.7], [0.2, 0.5], [0.6, 0.9])	([0.2, 0.7], [0.1, 0.4], [0.7, 0.9])	([0.3, 0.7], [0.6,0.8], [0.5, 0.7])
$\boldsymbol{\lambda(a)}$	([0.7,0.4, 0.1])	([0.4,0.3, 0.7])	([0.7,0.3,0.8])	([0.9,0.5,0.2])
Evaluation Point $\boldsymbol{\Xi_{\Lambda}(a)}$	([0.2, -0.1], [0.3, -0.1], [-0.3, 0.7])	([0.1, 0.3], [0.1, 0.2], [0.1, 0.2])	([0.5, 0.0], [0.2,0.1], [0.1, 0.1])	([0.6,- 0.2], [-0.1,0.3], [-0.3, 0.5])

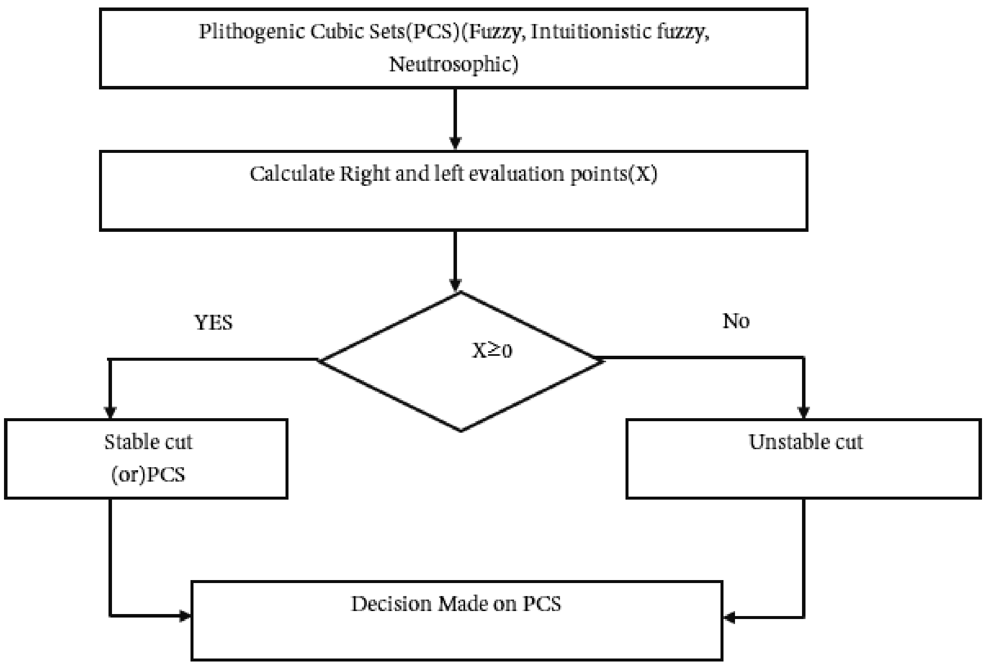

Fig. 1. Stable plithogenic cubic sets.

Theorem 1. The complement of an N-stable PNCS is also N-stable.

Proof: let $\Lambda =\langle Z,\lambda\rangle$ be a stable PNCS in A.

Then,

$A = SC_{\Lambda} = \{a \in A | L(E_{\Lambda}(A)) \geq 0,\ R(\Xi_{\Lambda}(A)) \geq 0\}$.

Hence $\lambda^N(a) - (Z^N(a))^- \geq 0$ and $Z(a)^+ - \lambda(a) \geq 0$ for all $a \in A$. It follows that $L(E_{\Lambda^c}(a)) = (1 - \lambda^N(a)) - (1 - (Z^N(a))^+) = (Z^N(a))^+ - \lambda^N(a)) \geq 0$ and

$$R(E_{\Lambda^c}(a)) = (1 - (Z^N(a))^-) - \left(1 - \lambda^N(a)\right) = \lambda^N(a)) - (Z^N(a))^- \geq 0.$$

Therefore $\Lambda^c = < Z^c, \lambda^c >$ is an N-stable PNCS.

The above theorem holds good for truth and falsity values.

Example 10. The following is an example to compute the complement for the stable PNCS with the values of an attribute (ransomware, adware, spyware, trojan horse) that represent the criteria "types of computer virus".

Table 10. Complement of $\Lambda = < Z, \lambda >$.

Dissimilarity Measure	0	0.50	0.75	1
Value of Attributes	Ransomware	Adware	Spyware	Trojan Horse
Appurtenance Measure $Z(a)$	([0.5, 0.7], [0.6, 0.8], [0.1, 0.5])	([0.2, 0.5], [0.1, 0.3], [0.3, 0.8])	([0.2, 0.6], [0.5, 0. 8], [0.3, 0.5])	([0.2, 0.7], [0.5,0.8], [0.3, 0.5)
$\lambda(a)$	([0.6,0.7, 0.3])	([0.3, 0.2, 0.5])	([0.7,0.7,0.4])	([0.5,0.7,0.4])
Evaluation Point $\Xi_{\Lambda}(a)$	([0.1, 0.1], [0.1, 0.1], [0.2, 0.3])	([0.1, 0.2], [0.2, 0.1], [0.2, 0.3])	([0.5, 0.1], [0.2, 0.1], [0.1, 0.1])	([0.3, 0.2], [0.2,0.1], [0.1, 0.1])
Appurtenance Measure $Z^c(a)$	([0.5, 0.3], [0.4, 0.2], [0.9, 0.5])	([0.8, 0.5], [0.9, 0.7], [0.7, 0.2])	([0.8, 0.4], [0.5, 0.2], [0.7, 0.5])	([0.8, 0.3], [0.5,0.2], [0.7, 0.5])
$\lambda^c(a)$	([0.4,0.3, 0.7])	([0.7, 0.8, 0.5])	([0.3,0.3,0.6])	([0.5,0.3,0.6])
Evaluation Point $\Xi^c{}_{\Lambda}(a)$	([0.3, 0.5], [0.1, 0.8], [0.1, 0.9])	([0.1, 0.2], [-0.1, 0.1], [0.2, 0.3])	([0.5, -0.1], [-0.3, 0.6], [-0.2, 0.4])	([0.3, 0.2], [-0.2,0.2], [-0.2, 0.4])

Theorem 2. The complement of an N-unstable PNCS is also N-unstable.

Proof: let $\Lambda = < Z, \lambda >$ be an N-unstable PNCS in A.

Then,

$UC_{\Lambda} = \{a \in A | L(E_{\Lambda}(a)) < 0\} \cup \{a \in A | R(E_{\Lambda}(a)) < 0\} \neq \varphi$,

and so there exist $a \in A$ such that $\lambda^N(a) - (Z^N(a))^- < 0$ or $(Z^N(a))^+ - \lambda^N(a) < 0$.

It follows that:

$$L(E_{\Lambda^c}(a)) = \left(1 - \lambda^N(a)\right) - (1 - (Z^N(a))^+) = (Z^N(a))^+ - \lambda^N(a)) < 0.$$

Or

$$R(E_{\Lambda^c}(a)) = (1 - (Z^N(a))^-) - \left(1 - \lambda^N(a)\right) = \lambda^N(a)) - (Z^N(a))^- < 0.$$

Hence $U_C{}^c \neq \varphi$, therefore, $\Lambda = < Z, \lambda >$ is an unstable PNCS in A.

The above theorem holds good for truth and falsity values.

Theorem 2. The P-union and P-intersection of two N-stable PNCSs in A are N-stable PNCSs in A.

Proof: let $\Lambda = < Z, \lambda >$ and $\theta = < K, \gamma >$ be N-stable PNCSs in A.

Then

$$SC_{\Lambda} = \{a \in A | L(E_{\Lambda}(A)) \geq 0,\ R(\Xi_{\Lambda}(A)) \geq 0\} = A,$$

and

$$SC_{\theta} = \{a \in A | L(E_{\theta}(A)) \geq 0,\ R(E_{\theta}(A)) \geq 0\} = A.$$

It follows that $\lambda^N(a) - (Z(a)^N)^- \geq 0$and $(Z(a)^N)^+ - \lambda^N(a) \geq 0$ for all $a \in A$ and

$\gamma^N(a) - (K(a)^N)^- \geq 0, (K(a)^N)^+ - \gamma^N(a) \geq 0$ for all $a \in A$.

Assume that $\lambda^N(a) \geq \gamma^N(a)$ and consider four cases:

I. $(\lambda^N(a))^- \geq (\gamma^N(a))^-$ and $(\lambda^N(a))^+ \geq (\gamma^N(a))^+$.

II. $(\lambda^N(a))^- \geq (\gamma^N(a))^-$ and $(\lambda^N(a))^+ \leq (\gamma^N(a))^+$.

III. $(\lambda^N(a))^- \leq (\gamma^N(a))^-$ and $(\lambda^N(a))^+ \geq (\gamma^N(a))^+$.

IV. $(\lambda^N(a))^- \leq (\gamma^N(a))^-$ and $(\lambda^N(a)^{)+} \leq (\gamma^N(a))^+$.

The first case implies that,

$$MAX\{\lambda^N(a), \gamma^N(a)\} = \lambda^N(a) \geq (Z^N(a))^- = MAX\{(Z^N(a))^-, (K^N(a))^-\},$$

And

$$MAX\{\lambda^N(a), \gamma^N(a)\} = \lambda^N(a) \leq (Z^N(a))^+ = MAX\{(Z^N(a))^+, (K^N(a))^+\}.$$

It follows that:

$$\lambda^N(a) - (Z^N(a))^- \geq 0 \text{ and } (Z^N(a))^+ - \lambda^N(a) \geq 0.$$

In the second case, we have

$$MAX\{\lambda^N(a), \gamma^N(a)\} = \lambda^N(a) \geq (Z^N(a))^- = MAX\{(Z^N(a))^-, (K^N(a))^-\},$$

And,

$$MAX\{\lambda^N(a), \gamma^N(a)\} = \lambda^N(a) \leq (Z^N(a))^+ = MAX\{(Z^N(a))^+, (K^N(a))^+\}.$$

It follows that:

$$\lambda^N(a) - (Z^N(a))^- \geq 0 \text{ and } (Z^N(a))^+ - \lambda^N(a) \geq (Z^N(a))^+ - \lambda^N(a) \geq 0.$$

The third case includes,

$$MAX\{\lambda^N(a), \gamma^N(a)\} = \lambda^N(a) \geq \gamma^N(a) \geq (K^N(a))^-$$
$$= MAX\{(Z^N(a))^-, (K^N(a))^-\} = MAX\{(Z^N(a))^-, (K^N(a))^-\}.$$

And,

$$MAX\{\lambda^N(a), \gamma^N(a)\} = \lambda^N(a) \leq (Z^N(a))^+ = MAX\{(Z^N(h))^+, (K^N(h))^+\}.$$

It follows that;

$$\lambda^N(a) - (K^N(a))^- \geq \gamma^N(a) - (K^N(a))^- \geq 0,$$

And,

$$(Z^N(a))^+ - \lambda^N(a) \geq 0.$$

For the final case,

$$MAX\{\lambda^N(a), \gamma^N(a)\} = \lambda^N(a) \geq (Z^N(a))^- = MAX\{(Z^N(a))^-, (K^N(a)^{)-}\},$$

And,

$$\mathrm{MAX}\{\lambda^{N}(a), \gamma^{N}(a)\} = \lambda^{N}(a) \leq (Z^{N}(a))^{+} \leq K^{N}(a) = \mathrm{MAX}\{(Z^{N}(a))^{+}, (K^{N}(a))^{+}\}.$$

It follows that:

$$\lambda^{N}(a) - (Z^{N}(a))^{-} \geq \gamma^{N}(a) - (K^{N}(a))^{+} \geq (K^{N}(a))^{+} - \gamma^{N}(a) \geq 0.$$

In this case $\gamma^{N}(a) \geq \lambda^{N}(a)$, we can obtain the same results similarly.

Therefore $\Lambda =< Z, \lambda >$, $\theta =< K, \gamma >$ are the N-stable PNCSs in A.

The above theorem holds good for truth and falsity values.

The following example supports the above theorem.

Example 11. Let us consider the attribute types of volcanoes with the attribute values cinder cones, composite volcanoes, shield volcanoes, and lava domes $\Lambda =< Z, \lambda >$, $\theta =< K, \gamma >$ be the PNCS in A with the *Table*s *11* and *12* correspondingly.

Table 11. $\boldsymbol{\Lambda =< Z, \lambda >}$.

Dissimilarity Measure	0	0.50	0.75	1
Value of Attributes	Cinder cones	Composite volcanoes	Shield volcanoes	Lava domes
Appurtenance Measure $\mathbf{Z(a)}$	([0.5, 0.6], [0.1, 0.4], [0.4, 0.8])	([0.3, 0.7], [0.2, 0.5], [0.6, 0.9])	([0.3, 0.7], [0.1, 0.7], [0.6, 0.9])	([0.1, 0.7], [0.5,0.8], [0.5, 0.7])
$\boldsymbol{\lambda(a)}$	([0.5,0.2, 0.6])	([0.4,0.3, 0.7])	([0.5,0.3,0.6])	([0.3,0.7,0.6])
Evaluation Point $\boldsymbol{\Xi_{\Lambda}(a)}$	([0.0, 0.1], [0.1, 0.2], [0.2, 0.2])	([0.1, 0.3], [0.1, 0.2], [0.1, 0.2])	([0.2, 0.2], [0.2, 0.4], [0.0, 0.3])	([0.2, 0.4], [0.2,0.1], [0.1, 0.1])

Table 12. $\boldsymbol{\theta =< K, \gamma >}$.

Dissimilarity Measure	0	0.5	0.75	1
Value of Attributes	Cinder cones	Composite volcanoes	Shield volcanoes	Lava domes
Appurtenance Measure $\mathbf{Z(a)}$	([0.1, 0.6], [0.5, 0.8], [0.4, 0.7])	([0.4, 0.8], [0.1, 0.5], [0.4, 0.9])	([0.1, 0.7], [0.1, 0.4], [0.6, 0.9])	([0.1, 0.7], [0.6,0.8], [0.5, 0.9])
$\boldsymbol{\lambda(a)}$	([0.5,0.6, 0.5])	([0.4,0.3, 0.7])	([0.5,0.3,0.8])	([0.4,0.7,0.6])
Evaluation Point $\boldsymbol{\Xi_{\Lambda}(a)}$	([0.4, 0.1], [0.1, 0.2], [0.1, 0.2])	([0.0, 0.4], [0.2, 0.2], [0.3, 0.2])	([0.4, 0.2], [0.2, 0.1], [0.2, 0.1])	([0.3,0.3], [0.1,0.1], [0.1, 0.3])

Table 13. $\boldsymbol{\Lambda \cup_{P} \theta = (Z \cup K, \lambda \vee \gamma)}$.

Dissimilarity Measure	0	0.50	0.75	1
Value of Attributes	Cinder cones	Composite volcanoes	Shield volcanoes	Lava domes
Appurtenance Measure $\mathbf{Z(a)}$	([0.1, 0.6], [0.5, 0.8], [0.4, 0.8])	([0.4, 0.8], [0.1, 0.5], [0.4, 0.9])	([0.1, 0.7], [0.1, 0.7], [0.6, 0.9])	([0.1, 0.7], [0.5,0.8], [0.5, 0.9])
$\boldsymbol{\lambda(a)}$	([0.5,0.6, 0.6])	([0.4,0.3, 0.7])	([0.5,0.3,0.8])	([0.4,0.7,0.6])
Evaluation Point $\boldsymbol{\Xi_{\Lambda}(a)}$	([0.4, 0.1], [0.1, 0.2], [0.2, 0.2])	([0.0, 0.4], [0.2, 0.2], [0.3, 0.2])	([0.4, 0.2], [0.2, 0.4], [0.2, 0.1])	([0.3, 0.3], [0.2,0.1], [0.1, 0.3])

Table 14. $\Lambda \cap_P \theta = (Z \cap K, \lambda \wedge \gamma)$

Dissimilarity Measure	0	0.50	0.75	1
Value of Attributes	Cinder cones	Composite volcanoes	Shield volcanoes	Lava domes
Appurtenance Measure $Z(a)$	([0.5, 0.6], [0.1, 0.4], [0.4, 0.7])	([0.3, 0.7], [0.2, 0.5], [0.6, 0.9])	([0.3, 0.7], [0.1, 0.4], [0.6, 0.9])	([0.1, 0.7], [0.6,0.8], [0.5, 0.7])
$\lambda(a)$	([0.5,0.2, 0.5])	([0.4,0.3, 0.7])	([0.5,0.3,0.6])	([0.3,0.7,0.6])
Evaluation Point $\Xi_\Lambda(a)$	([0.0, 0.1], [0.1, 0.2], [0.1, 0.2])	([0.1, 0.3], [0.1, 0.2], [0.1, 0.2])	([0.2, 0.2], [0.2, 0.1], [0.0, 0.3])	([0.2,0.4], [0.1,0.1], [0.1, 0.1])

Theorem 3. Let $\Lambda = < Z, \lambda >$ and $\theta = < K, \gamma >$ be N-internal PNCSs in A such that;

$$(\text{for all } a \in A)\left(MAX\{(Z(a)^N)^-, (K(a)^N)^-\} \leq (\lambda^N \wedge \gamma^N)(a)\right).$$

Then, the R-union of Λ and θ is an N-stable PNCS in A.

Proof: let $\Lambda = < Z, \lambda >$ and $\theta = < K, \gamma >$ be N- internal PNCSs in A.

Then,

$$(Z(a)^N)^- \leq \lambda^N(a) \leq (Z(a)^N)^+ \text{ and } (K(a)^N)^- \leq \lambda^N(a) \leq (K(a)^N)^+ \text{ for all } a \in A.$$

It follows that;

$$(\text{for all } a \in A)\left(MAX\{(Z(a)^N)^-, (K(a)^N)^-\} \leq (\lambda^N \wedge \gamma^N)(a)\right),$$

That,

$$MAX\{(Z(a)^N)^-, (K(a)^N)^-\} \leq (\lambda^N \wedge \gamma^N)(a) \leq MAX\{(Z(a)^N)^+, (K(a)^N)^+\} \text{ for all } a \in A.$$

Hence, the R-union of Λ and θ is an N-IPNCS is N-stable.

The above theorem holds good values for truth and falsity values.

Theorem 4. Let $\Lambda = < Z, \lambda >$ and $\theta = < K, \gamma >$ be N-internal PNCSs in A such that

$$(\text{for all } a \in A)\left(MAX\{(Z^N)^-(a), (K^N)^-(a)\} \leq (\lambda^N \vee \gamma^N)(a)\right).$$

Then, the R-intersection of Λ and θ is an N-stable PNCS in A.

Proof: let $\Lambda = < Z, \lambda >$ and $\theta = < K, \gamma >$ be N-internal PNCSs in A.

Then,

$$(Z^N(a))^- \leq \lambda^N(a) \leq (Z^N(a))^- \text{ and } (K^N(a))^- \leq \gamma^N(a) \leq (K^N(a))^+ \text{ for all } a \in A.$$

It follows that:

$$(\text{for all } a \in A)\left(MAX\{(Z^N(a))^-, (K^N(a))^-\} \leq (\lambda^N \vee \gamma^N)(a)\right),$$

That,

$$MAX\{(Z^N(a))^-, (K^N(a))^-\} \leq (\lambda^N \vee \gamma^N)(a) \leq MAX\{(Z^N(a))^+, (K^N(a))^+\} \text{ for all } a \in A.$$

Hence, the R-intersection of Λ and θ is an N-IPNCS, so it is N-stable.

The above theorem holds good values for truth and falsity.

The following example supports the above theorem.

Example 12. Let us consider the attribute values types of refrigerator compressors: reciprocating, rotary vane, screw, centrifugal, and $\Lambda =< \mathrm{Z}, \lambda >$, $\theta =< \mathrm{K}, \gamma >$ be the PNCS in A with the *Tables 15* and *16* correspondingly.

Table 15. $\boldsymbol{\Lambda =< Z, \lambda >}$.

Dissimilarity Measure	0	0.50	0.75	1
Value of Attributes	Reciprocating	Rotary vane	Screw	Centrifugal
Appurtenance Measure $\mathbf{Z(a)}$	([0.2, 0.7], [0.2, 0.4], [0.3, 0.7])	([0.1, 0.8], [0.2, 0.6], [0.7, 0.9])	([0.3, 0.7], [0.1, 0.6], [0.6, 0.9])	([0.1, 0.6], [0.5,0.8], [0.4, 0.7])
$\boldsymbol{\lambda(a)}$	([0.5,0.3, 0.6])	([0.4,0.3, 0.7])	([0.5,0.3,0.7])	([0.4,0.6,0.5])
Evaluation Point $\boldsymbol{\Xi_\Lambda(a)}$	([0.3, 0.2], [0.1, 0.1], [0.3, 0.1])	([0.3, 0.4], [0.1, 0.3], [0.0, 0.2])	([0.2, 0.2], [0.2, 0.3], [0.1, 0.2])	([0.3, 0.2], [0.1,0.2], [0.1, 0.2])

Table 16. $\boldsymbol{\theta =< K, \gamma >}$.

Dissimilarity Measure	**0**	0.50	0.75	1
Value of Attributes	Reciprocating	Rotary vane	Screw	Centrifugal
Appurtenance Measure $\mathbf{Z(a)}$	([0.2, 0.6], [0.2, 0.5], [0.4, 0.8])	([0.4, 0.7], [0.2, 0.6], [0.6, 0.8])	([0.2, 0.7], [0.1, 0.7], [0.6, 0.9])	([0.1, 0.5], [0.5,0.8], [0.5, 0.7])
$\boldsymbol{\lambda}$ **(a)**	([0.5,0.3, 0.6])	([0.5,0.3, 0.7])	([0.5,0.4,0.7])	([0.3,0.7,0.5])
Evaluation Point $\boldsymbol{\Xi_\theta(a)}$	([0.3, 0.1], [0.1, 0.2], [0.2, 0.2])	([0.1, 0.3], [0.1, 0.3], [0.1, 0.1])	([0.3, 0.2], [0.3, 0.3], [0.1, 0.2])	([0.2, 0.2], [0.2,0.1], [0.0, 0.2])

Table 17. $\boldsymbol{\Lambda \cup_R \theta = (Z \cup K, \lambda \wedge \gamma)}$.

Dissimilarity Measure	0	0.50	0.75	1
Value of Attributes	Reciprocating	Rotary vane	Screw	Centrifugal
Appurtenance Measure $\mathbf{Z(a)}$	([0.2, 0.7], [0.2, 0.5], [0.4, 0.8])	([0.1, 0.8], [0.2, 0.6], [0.7, 0.9])	([0.3, 0.7], [0.1, 0.7], [0.6, 0.9])	([0.1, 0.6], [0.5,0.8], [0.4, 0.7])
$\boldsymbol{\lambda}$ **(a)**	([0.5,0.3, 0.6])	([0.4,0.3, 0.7])	([0.5,0.3,0.7])	([0.3,0.6,0.5])
Evaluation Point $\boldsymbol{\Xi_\theta(a)}$	([0.3, 0.2], [0.1, 0.1], [0.2, 0.2])	([0.3, 0.4], [0.1, 0.3], [0.0, 0.2])	([0.2, 0.7], [0.2, 0.4], [0.1, 0.2])	([0.2, 0.3], [0.1,0.2], [0.1, 0.2])

Table 18. $\boldsymbol{\Lambda \cap_R \theta = (Z \cup K, \lambda \vee \gamma)}$.

Dissimilarity Measure	0	0.50	0.75	1
Value of Attributes	Reciprocating	Rotary vane	Screw	Centrifugal
Appurtenance Measure $\mathbf{Z(a)}$	([0.2, 0.6], [0.2, 0.4], [0.3, 0.7])	([0.4, 0.7], [0.2, 0.6], [0.6, 0.8])	([0.2, 0.7], [0.1, 0.6], [0.6, 0.9])	([0.1, 0.5], [0.5,0.8], [0.5, 0.7])
$\boldsymbol{\lambda}$ **(a)**	([0.5,0.3, 0.6])	([0.5,0.3, 0.7])	([0.5,0.4,0.7])	([0.4,0.7,0.5])
Evaluation Point $\boldsymbol{\Xi_\theta(a)}$	([0.3, 0.1], [0.1, 0.1], [0.3, 0.1])	([0.1, 0.2], [0.1, 0.3], [0.1, 0.1])	([0.3, 0.2], [0.3, 0.2], [0.1, 0.2])	([0.3, 0.1], [0.2,0.1], [0.0, 0.2])

Corollary 1. Let $\Lambda =< \mathrm{Z}, \lambda >$ and $\theta =< \mathrm{K}, \gamma >$ be N-EPNCS in A such that $\Lambda^* =< \mathrm{Z}, \gamma >$ and $\theta^* =< \mathrm{K}, \lambda >$ are N-IPNCS in A. Then the P-union $\Lambda \cup \theta$ and the P-intersection $\Lambda \cap \theta$ of $\Lambda =< \mathrm{Z}, \lambda >$ and $\theta =< \mathrm{K}, \gamma >$ are stable in A.

4 | Conclusion

In this research work, we have studied stable plithogenic cubic sets in which stable and unstable conditions for plithogenic fuzzy cubic, plithogenic intuitionistic fuzzy cubic, and plithogenic neutrosophic cubic, with

several numerical examples being discussed. The P and R order for stable and unstable plithoenic neutrosophic cubic sets and the core properties were introduced. In the future, we can extend the concept to almost stable plithogenic cubic sets and also apply the introduced concepts in multi-criteria decision-making.

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Author Contributions

S.P.Priyadharshini has contributed to the formulation of the mathematical models and theoretical framework. F.Nirmala Irudayam has been responsible for data analysis, computations and verifying the mathematical proofs. J.Ramkumar has focused on writing the manuscript, editing the content and providing revisions for intellectual content. All authors have approved the final version of the manuscript and agree with its submission.

Data Availability Statement

The data and mathematical computations supporting the findings of this study are available upon reasonable request from the corresponding author. All theoretical work has been included in the paper.

Conflicts of interest: the authors declare no conflict of interest.

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